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Steady-State Performance Variants of Non-Negative Least-Mean-Square Algorithm and Convergence Analysis

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Abstract: Due to the inherent physical characteristics of systems under investigation, non-negativity is one of the most interesting constraints that can usually be imposed on the parameters to estimate. The Non-Negative Least-Mean-Square algorithm (NNLMS) was proposed to adaptively find solutions of a typical Wiener filtering problem but with the side constraint that the resulting weights need to be non-negative. It has been shown to have good convergence properties. Nevertheless, certain practical applications may benefit from the use of modified versions of this algorithm. In this paper, we derive three variants of NNLMS. Each variant aims at improving the NNLMS performance regarding one of the following aspects: sensitivity of input power, unbalance of convergence rates for different weights and computational cost. We study the stochastic behavior of the adaptive weights for these three new algorithms for non-stationary environments. This study leads to analytical models to predict the first and second order moment behaviors of the weights for Gaussian inputs. Simulation results are presented to illustrate the performance of the new algorithms and the accuracy of the derived models.

Keywords: Non-Negative Least-Square (NNLS), Non-Negative Matrix Factorization (NMF), Non-Negative Least-Mean-Square (NNLMS).

I. INTRODUCTION

Optimization of a cost function given a set of constraints is a common objective in signal processing estimation problems. The constraints are usually imposed by system specifications which provide a priori information on the feasible set of solutions. The solution of estimation problems under constraints poses special problems for online applications. Common real-time signal processing restrictions on computational complexity and memory requirements tend to rule out several good solutions to the constrained optimization problem. Non-negativity is one of the most commonly stated constraints. It is often imposed on the parameters to estimate in order to avoid physically absurd and un interpretable results. Non-negativity constraints have been used for image de-blurring, de-convolution of system impulse response estimation and audio processing [3]. Another similar problem is the non-negative matrix factorization (NMF), which is now a popular dimension reduction technique used in many applications this problem is closely related to blind de-convolution, and has found direct application in neuroscience and in hyper spectral imaging Separation of non-negative mixture of non-negative sources has also been considered. Over the last fifteen years, a variety of methods have been developed to tackle non-negative least-square (NNLS) problems. Active set techniques for NNLS use the fact that if the set of variables which activate constraints is known, and then the solution of the constrained least-square problem can be obtained by solving an unconstrained one that includes only inactive variables.

The active set algorithm of Lawson and Hanson is a batch resolution technique for NNLS problems. It has become a standard among the most frequently used methods. In, Bro and De Jong introduced a modification of the latter, called Fast NNLS, which takes advantage of the special characteristics of iterative algorithms involving repeated use of non-negativity constraints. Projected gradient algorithms form another class, which is based on successive projections onto the feasible region. In, Lin used this kind of algorithm for NMF problems. Low memory requirements and simplicity make algorithms in this class attractive for large scale problems. Nevertheless, they are characterized by slow convergence rate if not combined with appropriate step size selection. The class of multiplicative algorithms is very popular for dealing with NMF problems. Particularly efficient updates were derived in this way for a large number of problems involving non-negativity constraints. However, these algorithms require batch processing, which is not suitable for online system identification problems.

In, the problem of online system identification under non-negativity constraints on the parameters to estimate was investigated. An LMS-type adaptive algorithm, called Non-Negative Least-Mean-Square (NNLMS) was proposed to solve the Wiener problem under the constraint that the resulting weights need to be non-negative. It was based on the stochastic gradient descent approach combined with a fixed point iteration which converges to a solution satisfying the Karush-Kuhn-Tucker conditions. The stochastic behavior of this algorithm was also analyzed in. In this paper, we extend the work of and derive useful variants of the NNLMS algorithm. Each of these variants is derived to improve the NNLMS properties in some sense. Normalized algorithm is proposed to reduce the NNLMS performance sensitivity to the input power value. An exponential algorithm is proposed to improve the balance of weight convergence rates as shown in Fig.1. Compared to NNLMS, the new algorithm leads to faster convergence of the weights in the active set (weights for which the inequality constraint is satisfied with the equal sign). Finally, a sign-based algorithm is proposed to reduce implementation cost in critical real-time applications.

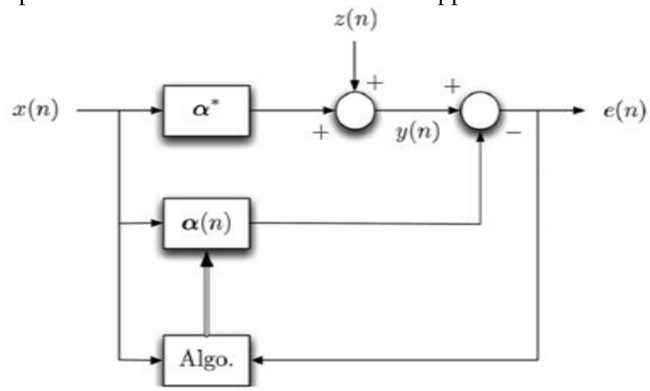

Fig.1. Adaptive system under study.

The rest of this paper is organized as follows. Section II reviews the system identification problem under non-negative constraints and the NNLMS algorithm. Section III motivates and introduces the NNLMS variants. In Sections IV and V, the transient behavior of each of these variants is analyzed. Analytical models are derived for the mean weight and for the mean-square error behavior. The accuracy of these models is illustrated through simulations. A final example compares the performance of the proposed algorithms with those of NLMS and Projected NLMS in solving an unconstrained non-negative parameter estimation problem. Each component in the update term of can be viewed as a distinct variable step size adjustment along the axis1. Hence, each component of will have a different convergence rate in general. Specifically in the case of weights in the active set (those that tend to zero in steady-state), the convergence rate will progressively reduce in time, becoming very small near steady-state. To alleviate this convergence rate unbalance, we introduce the Exponential NNLMS algorithm. To achieve a faster convergence for the adaptive coefficients as they get close to zero we propose the use of in (11), with parameter chosen in order to attract small values of towards zero. This leads to the th weight update equation.

II. VARIANTS OF THE NON-NEGATIVE LEAST-MEAN-SQUARE ALGORITHM

A. Normalized NNLMS

A direct extension of the original algorithm is the Normalized NNLMS. Conditioned on , the product in (13) has dimension of signal power. Thus, is inversely proportional to signal power. Hence, setting a constant value for leads to different weight updates for different signal power levels. This is the same sensitivity to signal power verified in the LMS algorithm. A popular way to address this limitation is to normalize the weight update by the input vector squared –norm which yields the Normalized NNLMS update equation

$$\boldsymbol{\alpha}_{\mathrm{N}}(n+1) = \boldsymbol{\alpha}_{\mathrm{N}}(n) + \frac{\eta}{\boldsymbol{x}^{\top}(n)\,\boldsymbol{x}(n)}\, e(n)\, \boldsymbol{D}_x(n)\, \boldsymbol{\alpha}_{\mathrm{N}}(n). \quad \textbf{(1)}$$

Like in Normalized LMS (NLMS) algorithm, adding a small positive regularization parameter to the denominator may be necessary to avoid numerical difficulties when becomes very small. The resulting –Normalized NNLMS will then be where we maintained the notation because (14) is a particular case of (15) for. From now on, we refer to simply as the Normalized NNLMS algorithm.

B. Exponential NNLMS

Each component in the update term of (13) can be viewed as a distinct variable step size adjustment along the axis1. Hence, each component of will have a different convergence rate in general. Specifically in the case of weights in the active set (those that tend to zero in steady-state), the convergence rate will progressively reduce in time, becoming very small near steady-state. To alleviate this convergence rate unbalance, we introduce the Exponential NNLMS algorithm. To achieve a faster convergence for the adaptive coefficients as they get close to zero we propose the use of in (11), with parameter chosen in order to attract small values of towards zero. This leads to the weight update equation (16) For, the weight update in (16) becomes larger than that in (13) when, thus accelerating convergence towards a null steady-state coefficient value. The condition for given can be easily determined from (16) as this condition, however, is not useful for design purposes, since it requires a priori knowledge of the algorithm behavior. We then propose a modified version of the update (16) that allows for instantaneous negative values of. The problem with real and negative instantaneous values of is that it may lead to a complex value for. To obtain always real values for we propose to use with and odd integers and. The oddness of and guarantees that. Then, the real solution for can be obtained by calculating. This leads to the following weight update equation for the Exponential NNLMS algorithm in vector form: with the component of defined as in the gamma correction used in image processing, an exponent in the range reduces the dynamic range of each. Large values of will be compressed towards 1 and small values of will be increased to prevent from stalling convergence. When, the update equation degenerates into the NNLMS algorithm (13). Using is generally not recommended, as it tends to spread the vector component values.

TABLE I: Computational Complexity

Algorithm	Recursion	Computational cost per iteration				Main property
		$+$	$\times$	sgn	$(\cdot)^{\gamma}$	
NNLMS	Equation (13)	$2N$	$3N+1$			Original one, simplicity
Norm. NNLMS	Equation (15)	$3N$	$4N+1$			Insensitivity to input power
Exp. NNLMS	Equation (18)	$2N$	$3N+1$		N	Balance on weight convergence
S-S NNLMS	Equation (19)	N	$2N$	N		Reduced computational cost

C. Sign-Sign NNLMS

Like Sign-Sign LMS, which has been included in the CCITT standard for adaptive differential pulse code modulation [24], the motivation for introducing a Sign-Sign NNLMS algorithm is its computational simplicity and its robustness against disturbances [25]. Replacing the input regressor vector and the estimation error in the update term with their signs reduces computation time and dynamic range requirements by replacing multiplications with shifts in real-time implementations. The Sign- Sign NNLMS algorithm is given by (19) after the two signs are evaluated, the component update given by where the sign before is determined by. The step-size is usually selected as a power of, say for some integer. In this case, (20) can be efficiently implemented using shift-add operations. Moreover, the non-negativity constraint will be always satisfied if is initialized with a positive vector and. Table I compares the computation complexities of NNLMS and its three variants described above. The rightmost column describes the anticipated property of each algorithm, to be verified in the following.

III. MEAN WEIGHT BEHAVIOR

Convergence in the mean sense of the NNLMS algorithm (13) has been studied in [20] for a stationary environment. We now study the stochastic behavior of the NNLMS variants introduced in Section III for fixed step sizes and for a time variant unconstrained solution given by where is a deterministic time-variant mean and is zero-mean with covariance matrix and independent of any other signal. This simple model provides some information on how the performances of the proposed algorithms are affected by a time variant optimal solution which consists of a deterministic trajectory and a random perturbation. The analysis using more elaborate non-stationarity models such as the random walk model or the autoregressive model [26] leads to mathematically intractable situations. This is due to the extra multiplication of the weight update term by a function of in as compared to the LMS algorithm. For the random walk model, the recursive equation for the covariance matrix of the adaptive weight vector becomes a function of the optimal weight covariance matrix, which becomes unbounded as time progresses [26]. For the autoregressive model, a nonlinear term given by the product of the weight error vector and the optimal weight update makes it impossible to determine a recursive adaptive weight vector covariance matrix equation in the state-space form [27].

The model (21) leads to a tractable analysis and still permits inferences about the behavior of the algorithms in randomly time variant environments by varying the power of. Inferences on the ability of the algorithm to track mean weight variations are also possible but require a different model run for each type of mean time variation of to be investigated. To conserve space and to simplify notation without ambiguity, from now on we use the generic notations and whenever the given expression is valid for all the algorithms under study. Notations, and will be used only for expressions which are specific to the corresponding algorithm. The same notational observation applies to any vector or matrix when referring to a specific algorithm. For the analyses that follow, we shall define the weight error vector with respect to the unconstrained solution as and the weight error vector with respect to the mean unconstrained solution as the two vectors are related by.

A. Statistical Assumptions

The following analysis is performed for and zero-mean stationary Gaussian and for white and statistically independent of any other signal. We assume in the subsequent mean weight behavior analysis that the input and weight vectors are statistically independent, according to the Independence Assumption. This assumption is typical in the study of adaptive algorithms. It is sometimes used for simplification and frequently required for mathematical tractability. The simulation results will show that the resulting analytical models have low as shown in Fig.2.

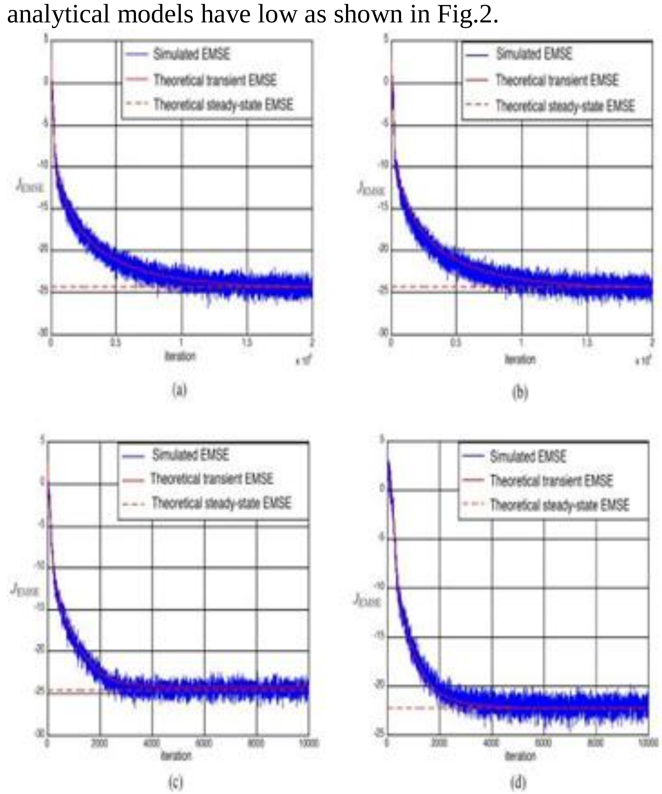

Fig.2. **Simulation Results.**

IV. SIMULATION RESULTS AND DISCUSSION

We now present simulation examples to illustrate the properties of the three algorithms and the accuracy of the models derived in Sections IV and V. The parameters for these examples were chosen to illustrate several properties of the three algorithms while conserving space. Similar results have been obtained using a variety of parameter sets. For all examples, . The unknown stationary system is defined as

$$\alpha_{o_i}^{*(\text{stat.})} = \begin{cases} 0.9 - 0.05\, i, & i = 0, \ldots, 18 \\ -0.01\,(i - 18) & i = 19, \ldots, 31. \end{cases}$$

For the non stationary case, we consider an unknown responder defined by

$$\alpha_{o_i}^{*(\text{nonstat.})}(n) = \boldsymbol{\alpha}_{o_i}^{*(\text{stat.})} + \frac{|\boldsymbol{\alpha}_{o_i}^{*(\text{stat.})}|}{10} \sin\left(\frac{2\,\pi}{T}\, n + 2\,\pi \frac{i-1}{N}\right) + \xi_i(n \quad \textbf{(2)}$$

V. CONCLUSION

Many real-life systems require non-negative coefficients when their physical behavior is parameterized. In such cases, a non-negativity constraint should be imposed on the parameters to estimate in order to avoid physically absurd and uninterpretable results. The Non-Negative Least-Mean-Square (NNLMS) algorithm has been recently proposed to solve such a constrained Wiener problem online. In this paper, we proposed three variants of NNLMS, each addressing a different issue that may affect NNLMS under given circumstances. The performances of the Normalized NNLMS, Exponential NNLMS and Sign-Sign NNLMS algorithms were studied for nonstationary environments. The optimal unconstrained solution was modeled by a time-variant mean plus a random fluctuation. The derived analytical models were shown to accurately predict both the mean and the mean-square behavior of the algorithms. Their performances were compared and their advantages in potential applications discussed. Future research efforts will further explore these NNLMS variants properties and apply them in practical situations where efficient adaptive solutions to non-negative filtering problem are required.

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