Abstract: This paper provides a comprehensive overview of critical developments in the field of multiple-input multiple-output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system. MIMO-OFDM channel estimation schemes play a very important role in achieving this aim. Structure of MIMO system increases the complexity of the estimator exponentially. This causes an increase in the computational burden of the transceivers. As a result, the complexity of the channel estimators is becoming an important issue in real world MIMO-OFDM applications.

In this paper, the performance of the low complexity Minimum Mean Square Error (MMSE) channel estimator scheme based on Karhunen-Loève (KL) series expansion coefficients for the 3GPP-LTE Downlink MIMO-OFDM systems is examined. System level simulations are accomplished to compare the performances of the estimators under the spatially correlated channel coefficient variations.

Keywords: 3GPP, LTE, MIMO, OFDM, MMSE, Karhunen-Loève.

I. INTRODUCTION

OFDM is a high performance candidate for wireless communication systems owing to its many advantages, especially its performance in frequency-selective fading channels. Besides that, MIMO transceiver architecture has potential to improve the system capacity. Hence in 3GPP-LTE, combined MIMOOFDM structure is anticipated to meet the demands of rapidly increasing number of applications on wireless mobile networks. In MIMO-OFDM systems channel state information is essential for the detection and equalization. In [1], 3GPP-LTE physical channels are described in detailed and modulation types are standardized. Besides that spatial channel models for MIMO simulations are described elaborately in [2]. In the time domain pilot-aided channel estimation, unoccupied subcarriers namely “guard bands”, adversely affect the performance of the estimator [3]. In order to reduce the complexity of the estimators, MMSE estimation of the KL series expansion coefficients is examined in [4]–[6]. In this paper, we compare the performances of the MMSE estimations of the KL series expansion coefficients of the 3GPP-LTE MIMO channel. In the simulations, the performances of the MMSE estimator of the KL expansion and LS estimator under the different channel correlations are compared. This paper provides a comprehensive overview of critical developments in the field of multiple-input multiple-output-Orthogonal Frequency Division Multiplexing (MIMO-OFDM) is a promising technique for reaching high data rates targeted in the 3rd Generation Partnership Project - Long Term Evolution (3GPP-LTE). MIMO-OFDM channel estimation schemes play a very important role on achieving this aim. Structure of MIMO system increases the complexity of the estimator exponentially.

II. OVERVIEW OF LTE DOWNLINK SYSTEM

According to, the duration of one frame in LTE Downlink system is 10 ms. Each LTE radio frame is divided into 10 sub-frames of 1 ms. As described in Fig.1 each sub-frame is divided into two time slots, each with duration of 0.5 ms. Each time slot consists of either 7 or 6 OFDM symbols depending on the length of the CP (normal or extended). In LTE Downlink physical layer, 12 consecutive subcarriers are grouped into one Physical Resource Block (PRB). A PRB has the duration of 1 time slot. Within one PRB, there are 84resource Elements(12subcarriers×7OFDM symbols) for normal CP or 72 resource elements (12subcarriers×6OFDM symbols)for extended CP.

![Fig. 1. 3GPP LTE downlink system model.](image)

LTE provides scalable bandwidth from 1.4 MHz to 20 MHz and supports both frequency division duplexing (FDD)
and time-division duplexing (TDD). Table 1 shows the different transmission parameters for LTE Downlink systems.

### TABLE I: LTE Downlink Parameters

<table>
<thead>
<tr>
<th>Transmission bandwidth (MHz)</th>
<th>1.25</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-frame duration (ms)</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-carrier spacing (KHz)</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling frequency (MHz)</td>
<td>1.92</td>
<td>3.84</td>
<td>7.68</td>
<td>15.36</td>
<td>23.04</td>
<td>30.72</td>
</tr>
<tr>
<td>FFT size</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td>Number of occupied sub-carrier</td>
<td>76</td>
<td>151</td>
<td>301</td>
<td>601</td>
<td>901</td>
<td>1201</td>
</tr>
</tbody>
</table>

#### A. System Model

The system model is given in Fig.2. A MIMO-OFDM system with \( N_{Tx} \) transmit and \( N_{Rx} \) receive antennas is assumed.

![Fig.2. MIMO-OFDM system.](image)

OFDMA is employed as the multiplexing scheme in the LTE Downlink systems. OFDMA is a multiple user’s radio access technique based on OFDM technique. OFDM consists in dividing the transmission bandwidth into several orthogonal sub-carriers. The entire set of subcarriers is shared between different users.

#### B. Signal Model

The block diagram of the simplified 3GPP-LTE, MIMOOFDM transmitter system with NT transmit antennas is shown in Fig.3 M-QAM modulated serial data stream is grouped into \( N_s \) sized blocks. A number of modulated \( N_s \) reference signals, namely pilots and \( N_d \) sized data signals are allocated to the resources elements as defined in [1]. Reference and data signals have been paralleled into consecutive data blocks in order to constitute an OFDM symbol. At the \( p \)th transmit antenna, the \( q \)th OFDM symbol can be represented as

\[
A_{pq}^T = [A_{pq}^T[0], A_{pq}^T[1], ..., A_{pq}^T[N_{y}-1]]^T \in \mathbb{C}^{N_{y} \times 1}
\]

![Fig.3. MIMO-OFDM Transmitter scheme for LTE Downlink.](image)

Where \( \Xi \) denotes the modulation alphabet and \( N_{y} = N_{d} + N_{p} \) is the number of the occupied subcarriers at each of the transmit antenna. Before the N-point Inverse Fast Fourier Transform (IFFT) block, in order to avoid interference, \( N_{GB} \) zeros are padded to the unused subcarriers at the edges of the spectrum as guard bands, where \( N_{GB} \) is equal to \( N - N_{y} \). Only the specified OFDM symbols contain the reference signal for the channel estimation. Considering only the reference signal inserted OFDM symbols in the data-aided channel estimation, the OFDM symbol indicator \( \ell \) could be omitted for the sake of simplicity. Reference and data signals with guard band before the IFFT block can be represented by \( X^{(p)} \in \mathbb{C}^{N_{y}} \) as below

\[
X^{(p)} = [A_{q_{eff}}^T, 0_{N_{y}-N_{y}}^T, A_{right}^T]^T
\]

Where

\[
A_{q_{eff}}^T = A^{(p)} \left[ \frac{N_{y}}{2}, \ldots, A^{(p)}[N_{y}-1] \right]^T
\]

\[
P_{right}^T = [A^{(p)}[0], A^{(p)}[1], \ldots, A^{(p)} \left[ \frac{N_{y}}{2} - 1 \right]]^T
\]

N-point IFFT block is derived by the OFDM symbol \( X^{(p)} \) to generate time domain transmitted signal \( X^{(p)} \) is found to be

\[
X^{(p)} = FX^{(p)}
\]

Where

\[
F = \begin{bmatrix}
1 & 1 & e^{j2\pi/N} & e^{j2\pi(1-L)/N} & \cdots & e^{j2\pi(N-1L)/N} \\
1 & e^{j2\pi/N} & e^{j2\pi(1-N)/N} & \cdots & e^{j2\pi(2-N)/N} & \cdots & e^{j2\pi((N-1)L-N)/N} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
1 & e^{j2\pi(N-1L)/N} & e^{j2\pi((N-2)L-N)/N} & \cdots & e^{j2\pi((N-1)L-N)/N} & \cdots & e^{j2\pi((N-1)L-N)/N}
\end{bmatrix}
\]

is the DFT matrix. In order to get rid of the inter-symbol interference, Cyclic Prefix (CP) with a size of \( N_{CP} \leq L \) is added to the signal \( X^{(p)} \) to obtain the time domain transmitted signal \( s^{(p)} \in \mathbb{C}^{N_{y}+N_{CP}} \). \( L \) represents the channel impulse response (CIR) vector length of the MIMO channel between the \( p \)th transmit antenna and the \( q \)th receive antenna where it can be expressed as \( h^{(pq)} = [h_{w}^{(q)}, h_{w}^{(q)}[1], \ldots, h_{w}^{(q)}[L-1]] \in \mathbb{C}^{L} \). In Fig.4, MIMO-OFDM receiver scheme with \( N_{ant} \) antennas is depicted. At each receive antenna, the signal \( y^{(q)} \in \mathbb{C}^{N_{y}+N_{CP}+L-1} \) can be expressed as the sum of the convolved signals \( s^{(p)} \) and \( h^{(pq)} \) from the transmit antennas. Before the FFT block at each receiver, unnecessary terms are removed and the signal combined into the subsequent blocks with a length of \( N \) termed as \( y^{(p,q)} \).

![Fig.4. MIMO-OFDM Receiver scheme for LTE Downlink.](image)

In the light of above time domain expressions, the frequency domain representation of the received signal at the \( q \)th antenna

\[
y^{(q)} = [y^{(q)}[0], y^{(q)}[1], ..., y^{(q)}[N-1]]^T \in \mathbb{C}^{N}\text{can be expressed as}
\]

\[
y^{(q)} = \Xi H^{(q)} + W_{r}^{(q)}
\]

where \( W_{r}^{(q)} \in \mathbb{C}^{N} \) is the additive white gaussian noise (AWGN) at the \( q \)th receive antenna with zero mean and covariance matrix \( C_{w} = \mathbb{E}[W_{r}^{(q)} W_{r}^{(q)\dagger}] = \sigma^{2} I_{N} \). \( X \) is the transmitted signal from all of the antennas in the frequency domain and can be represented as below

\[
\Xi = [X_{diag}^{(q)}, X_{diag}^{(2)}, ..., X_{diag}^{(N_{y})}] \in \mathbb{C}^{N_{y} \times N_{y}}
\]

where \( X_{diag}^{(p)} = \text{DIAG}[X^{(p)}] \in \mathbb{C}^{N_{y} \times N} \) and \( p = 1, 2, ..., N_{y} \). \( H^{(q)} \in \mathbb{C}^{N_{y} \times N} \) is the combined channel frequency response (CFR) from all of the transmit antennas to the \( q \)th receive antenna, expressed by

\[
H^{(q)} = [H^{(q_{1})}_{T}, H^{(q_{2})}_{T}, \ldots, H^{(q_{N_{y}})_{T}}]^T
\]
An Efficient Channel Estimation Approach based on MMSE Estimator for Downlink MIMO-OFDM System

where $\Theta \in C^{N_p \times N_p}$ can be obtained as $\Theta = \mathbf{I}_{N_p} \otimes \mathbf{X}$. In LTE MIMO structure, the reference signals are orthogonal to each other. In Fig. 5, a simple reference signal orthogonality is depicted. Thus $(\Theta^\mathbb{H} \Theta)^{-1}$ in (16) can be easily derived as in [6] below

$$\hat{H}_s = \Theta^\mathbb{H} \hat{\mathbf{Y}}$$  \hspace{1cm} (18)

III. CHANNEL ESTIMATION

In this section, LS channel estimation and MMSE estimation of the Karhunen-Lo`eve expansion coefficients will be derived. Assume that only the symbols which contain the reference signal are selected. In the frequency domain signal representation, there exist $N_r$ reference signals.

Denoting

$$\hat{Y} = \left[Y^{(1)}_r, Y^{(2)}_r, ..., Y^{(N_r)}_r\right]^\mathbb{H}$$

as the received signal vector $Y \in \mathbb{C}^{N_r \times 1}$, the whole signal model can be written as in [5]:

$$Y = \left(I_{N_r} \otimes \mathbf{X}\right) H_s + W$$ \hspace{1cm} (14)

where $W \in \mathbb{C}^{N_r \times 1}$ is the zero mean white Gaussian noise with a covariance matrix of $\mathbb{C}_w = E[WW^\mathbb{H}] = \sigma^2 I_{N_r \times N_r}$.

A. Least Squares Estimator

The LS estimator of the reference signals can be derived as

$$\hat{H}_s = (\Theta^\mathbb{H} \Theta)^{-1} \Theta^\mathbb{H} \hat{\mathbf{Y}}$$  \hspace{1cm} (17)

B. Karhunen-Lo`eve Expansion

The frequency response of the MIMO channel at reference signal locations $\hat{H}_s$ is a zero mean random variable with covariance matrix $\mathbb{C}_\mathbf{H}_s$. KL transformation makes the channel vector orthogonal such that it can be represented by KL coefficients basis vectors as below

$$\hat{H}_s = \sum_{i=1}^{N_p} g_i \psi_i = \Psi g$$  \hspace{1cm} (19)

where $\psi_i$’s are orthonormal basis vectors constituting $\Psi = [\psi_1, \psi_2, ..., \psi_{N_p}]$ and $g_i$’s are the KL expansion coefficients defined in vector $g = [g_1, g_2, ..., g_{N_p}]^\mathbb{H}$. Consequently defining $E[gg^\mathbb{H}] = \Lambda_g$, the channel covariance matrix at reference signal positions can be easily expressed as

$$\mathbb{C}_\mathbf{H}_s = E[\hat{H}_s(\hat{H}_s)^\mathbb{H}] = \Psi \Lambda_g \Psi^\mathbb{H}$$  \hspace{1cm} (20)

In the above equation, if the $\Lambda_g$ matrix is diagonal, $\Psi \Lambda_g \Psi^\mathbb{H}$ expression becomes the Singular Value Decomposition of the $\mathbb{C}_\mathbf{H}_s$ matrix. Thus we can express (15) in a different form

$$\hat{Y} = \Theta \Psi g + \hat{W}$$  \hspace{1cm} (21)

Finally, the MMSE estimator of the KL expansion coefficients can be expressed as in [5]:

$$\hat{g} = \Lambda_g^{-1} \left(\Lambda_g + \sigma^2 I_{N_p}\right)^{-1} \Psi^\mathbb{H} \Theta^\mathbb{H} \hat{Y}$$

$$= \Gamma^\mathbb{H} \Theta^\mathbb{H} \hat{Y}$$  \hspace{1cm} (22)

$\Gamma$ can be denoted as

$$\Gamma = \Lambda_g^{-1} \left(\Lambda_g + \sigma^2 I_{N_p}\right)^{-1} = \text{diag} \left\{ \lambda_{g_1}, ..., \lambda_{g_{N_p}} \right\}$$  \hspace{1cm} (23)

where $\lambda_{g_1}, \lambda_{g_2}, ..., \lambda_{g_{N_p}}$’s as the singular values of the $\Lambda_g$. Assuming the rank of the matrix $\Lambda_g$ as, the MMSE estimator of the optimum truncated KL expansion can be defined as

$$\Gamma_{\text{tr}} = \text{diag} \left\{ \frac{\lambda_{g_1}}{\lambda_{g_1} + \sigma^2}, ..., \frac{\lambda_{g_{N_p}}}{\lambda_{g_{N_p}} + \sigma^2} \right\}$$  \hspace{1cm} (24)
As a result, the MMSE estimator of the KL expansion coefficient requires only the division operations of the coefficients instead of huge matrix multiplications.

IV. EXPERIMENTAL RESULTS
This section gives the details about the performance evaluation of the proposed approach. The Bit error rate (BER) is evaluated to verify the performance. The comparative analysis was also done in this paper to show the effectiveness of proposed approach.

The above figure BER results for estimated and perfect channel state information. In Fig.6; the performances of the LS estimator, optimal rank and reduced rank truncated KL expansion MMSE estimators are compared in Bit Error Rate (BER) sense. There exist an error floor for the reduced rank KL MMSE estimator greater than the values of a 20 dB Eb/N0. The fig.7 shows the BER curves for the LTE 2x2 MIMO-OFDM system with different correlation coefficients. In Fig.7. BER results are examined according to different correlation coefficients for the same LTE spatial channel model [7].

![Fig.6. BER results for estimated and perfect channel state information.](image)

![Fig.7. BER curves for the LTE 2x2 MIMO-OFDM system with different correlation coefficients.](image)

V. CONCLUSION
The KL expansion MMSE estimator exhibits a better performance than the LS estimator in the view of BER criteria. Reduced rank truncated KL expansion coefficient estimator performance is also better than the LS at low Eb/N0 values. However at high Eb/N0 values LS estimator exhibits a higher performance than the truncated KL expansion estimator. In addition to increased performance, KL expansion MMSE estimator is also a computationally efficient structure for the LTE downlink MIMO-OFDM systems.

VI. REFERENCES