Optimum Solution for Weighted Sum Rate Maximization in Gaussian Broadcast

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ABSTRACT- *In wireless communications resource allocation is the very most important task that has to dine at base station. In this paper we propose an efficient resource allocation scheme which maximizes the weighted rate sum (MWSR) in the Gaussian Broadcast Channels. Here we propose an efficient solution for MWSR (Maximum Weighted Sum Rate) which is in the closed form and need iterate again. Thus our proposed method reduces the number of iterations in power allocations and very easy to implement in real communications.*

***Keywords** - MWSR (Weighted Rate Sum Rate), Gaussian broadcast channel, power allocation, closed-form solution.*

I. INTRODUCTION

In recent years, the bandwidth or the capacity region of broad cast channels of the MIMO (MIMO BC) has attracted more attention of the researchers. Especially the broadcast channels of the MIMO are non degraded BCs and all these are well defined in terms of capacity region. Weingarten has showed that even the DPC (Dirty Paper Coding) has occupies the total capacity region of MIMO Broadcast Channels. However with some power constraints it is possible to transform the non-convex broad cast channel of MIMO to dual MIMO MAC (MIMO multiple access channel (MIMO-MAC). However in wireless communication systems the Gaussian Broadcast channel is considered as the basic downlink channel and it is belongs to the family of degraded MIMO BC. The capacity of the any MIMO BC is very important factor in the case of data transmissions and the boundaries of the broadcast channels a vital role in the capacity regions. However the capacity regions of the channels are described by using the concept of weighted rate sum maximization. In real scenario the quality of the service that a user will get is depends on the boundaries of the capacity regions of the broadcast channels and these boundaries are different for different end users, and these different boundaries for different customers through a common channel is achieved by using *weighted rate sum maximization technique.* Here by varying the weights the different boundaries of the broadcast channels is achieved.

Therefore for providing the efficient fairness QoS to all the users it is compulsory to solve the weighted rate sum maximization. In simple words for providing the faired quality of services to the users, it is compulsory to determine the boundaries of the broadcast channel by using weights.

An iterative procedure *Water Filling Optimal Power Allocation* is proposed for solving the weighted rate sum maximization problem which is a very complex one and a time consuming process since it consists of several iterations and similar an another approach is proposed for solving the weighted rate sum maximization known as greedy algorithm based on Lagrangian multiplier method both these approaches not efficient in dealing with multiple antenna Gaussian broadcast channels and it is hard to implement in real time practices and these are not dynamic..

Therefore in this paper we proposed a systematic method for solving the weighted rate sum maximization and we propose a closed-form optimal power allocation in single – antenna Gaussian broadcast channel. When compared to the previous approaches our new proposed method provide best solution for weighted rate sum maximization problem in one time and it does not requires water levels or iteration processes. Another advantage with this is, it is especially designed for dynamic power allocations.

The remaining of this article is arranged as, section II describes about the considered system model. Section III consists the problem formulation and correspondingly the solution for objective function is discussed in section IV. Finally the results analyzed in section V.

2. SYSTEM MODEL

Consider a single transmitting and multiple receiving communication system as shown in figure 1. Assume that the transmitter sends the information X to K receivers through k individual Gaussian broadcast channels. And each and every receiver is assumed that suffers from fading. Therefore the received signal is the addition of Gaussian Noise 'Z_k' to the corresponding transmitted signal through the channel multiplied by a channel gain 'g_k'

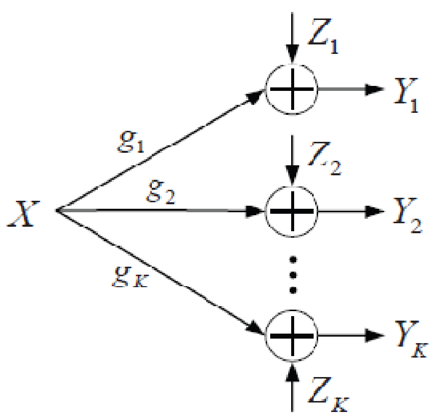

Fig.1: The Gaussian broadcast channel

Therefore Gaussian broadcast channel is modeled as

$$Y_k = g_k X + Z_k, k = 1,2,3\cdots k \qquad (1)$$

Z_k: Gaussian noise experienced by k^{th} receiver Equation 1 can be transformed into standard form by using elementary transformation function as

$$Y_k^{/} = X + Z_k^{/}, k = 1,2,3\cdots k \qquad (2)$$

$$Y_k^{/} = \frac{Y_k}{g_k}$$

$$Z_k^{/} = \frac{Z_k}{g_k}$$

$$N_k = \frac{\sigma^2{}_k}{g^2{}_k}$$

Therefore the Gaussian broadcast channel (2) capacity region is defined as [1]-[4]

$$C_{BC} = \left\{ (R_1, R_2, \cdots R_k) \middle| R_k \le \frac{1}{2}\log(1+\frac{\gamma_k P}{N_k + \sum_{l \le k}\gamma_l P} \right\} \qquad (3)$$

P: Transmit power.

γ_k : Proportion of the transmit _ power allocation to the k^{th} receiver,

3. PROBLEM FORMULATION

Consider weighted rate sum maximization problem as

$$\max_{\gamma_1,\gamma_2\cdots\gamma_k} \sum_{k=1}^{K} \omega_k R_k, (R_1, R_2, \cdots R_K) \in C_{BC} \qquad (4)$$

ω_k : k^{th} receiver weight.

Therefore the maximization problem of above considered weighted rate sum maximization problem is

$$\max_{\gamma_1,\gamma_2\cdots\gamma_k} \frac{\omega_1}{2}\log(1+\frac{\gamma_1 P}{N_1}) + \frac{\omega_2}{2}\log(1+\frac{\gamma_2 P}{N_2}) \cdots \frac{\omega_k}{2}\log(1+\frac{\gamma_k P}{N_k})$$
$$\text{where}, 0 \le \gamma_k \le 1, k = 1,2,3,\cdots k$$

(5)

Equation (5) is required objective problem formulation and it shows that it is a non convex in nature.

In the next session we find out a closed-form optimal solution for our non convexed above objective problem (5).

4. THE CLOSED-FORM SOLUTION OF WEIGHTED RATE SUM MAXIMIZATION

From the equation 5, the maximization problem can be written by substituting $\gamma_k = 1 - \sum_{k=1}^{k-1}\gamma_k$

$$\max_{\gamma_1,\gamma_2\cdots\gamma_k} f(\gamma_1,\gamma_2,\cdots\gamma_{k-1})$$
$$= \frac{1}{2}\log(\frac{(N_k+P)^{\omega_k}}{(N_1)^{\omega_1}}) + \frac{1}{2}\log(\frac{(N_k+\gamma_1 P)^{\omega_1}}{(N_2+\gamma_1 P)^{\omega_2}}) + \cdots$$
$$+ \frac{1}{2}\log(\frac{(N_{k-1}+\sum_{k=1}^{K-1}\gamma_1 P)^{\omega_{k-1}}}{(N_k+\sum_{k=1}^{K-1}\gamma_1 P)^{\omega_k}})$$

(6)

Above equation exploits some special properties as For any positive real constants $N_m, N_n, \omega_m, \omega_n, P when N_m < N_n$

$$g_{m,n}(x) = \frac{1}{2}\log\frac{(xP+N_m)^{\omega_m}}{(xP+N_m)^{\omega_n}}$$

Has the values of [0, 1]

Therefore based on above property the solution of the optimal power allocation for the maximization problem of (6) is

$$\gamma_1^* = \min\{\beta_{1,2,}\beta_{1,}2\cdots\beta_{1,}K\},$$
$$\gamma_2^* = \min\{[\beta_{2,3}-\gamma_1^*]^{\mp}[\beta_{2,4}-\gamma_1^*]^{\mp}\cdots[\beta_{1,}K\gamma_1^*]^{\mp}\}$$
$$\cdots$$
$$\gamma_{k-2}^* = \min\left\{[\beta_{k-2,k-1}-\sum_{l=1}^{k-3}\gamma_1^*]^{\mp}[\beta_{k-2,k}-\sum_{l=1}^{k-3}\gamma_1^*]\right\}$$
$$\gamma_{k-1}^* = [\beta_{k-1,k}-\sum_{l=1}^{k-2}\gamma_1^*]^{\mp}$$

Where

$$\beta_{m,n} = \begin{cases} 1, \omega_m \geq \omega_n \\ \left[\dfrac{\omega_m N_n - \omega_n N_m}{(\omega_n-\omega_m)}\right]^{\mp} \end{cases}$$

5. RESULTS AND ANALYSIS

Initially the proposed closed-form optimal solution for the problem of weighted sum rate maximization is showed in below figure 2 as a comparison between theoretical and simulation results. As discussed in equations 1-3 the sum rate is the linear addition of the individual users and the below results show same.

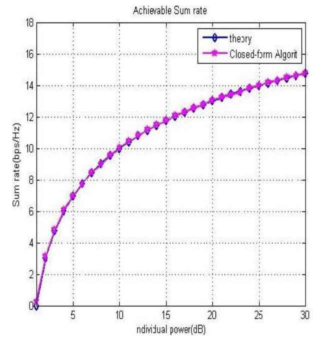

Fig2: Simulation results of closed form algorithm

Our proposed closed form solution for weighted rate sum maximization is first applied to the Gaussian BC with one and two users. Thus the weighted rate for two individual users is shown in below figure 3.

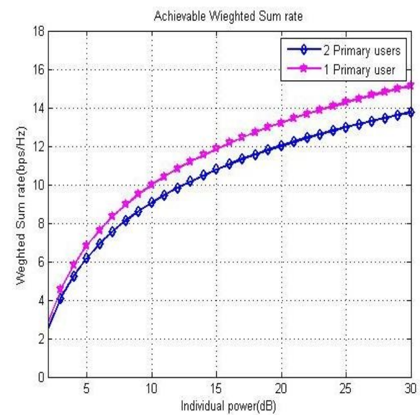

Fig3: Achieved weighted sun rate for two individual users.

From above figure, as discussed already in equation 1 he achieved weighted sum for single user is greater than the two users, since I the case of single the whole capacity region of the BC is assigned only to the single user but in case of two users the available capacity region of the Gaussian broadcast channel is assigned to two individual users which causes to reduces the weighted sum.

However the performance of the communication system is measured in terms of Bit error Rate. The performance of the considered system is shown in figure 4

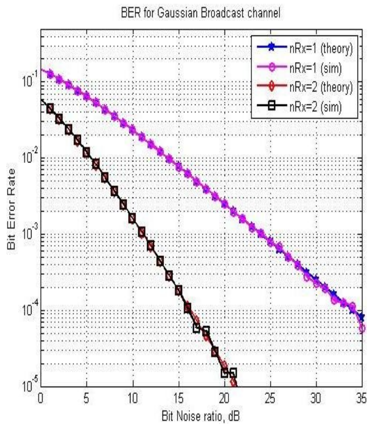

Fig4: performance results

From the above results, we proved that the bit error rate for the two receiving antenna is better the a single receiving antenna. Since the probability of loss of information packets is very less while we are using two individual antennas for data receiving.

Thus we successfully implemented our proposed closed form solution in our considered communication system and obtain the better results than before.

6. CONCLUSIONS

In this paper we described an optimal closed form solution for weighted rate sum maximization which is very simple and no need to iterate again. The proposed closed-form optimum solution may adoptable higher degree antenna systems also where we only proved up to two degree antenna system.

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